

Bulk antisymmetric tensor fields in a Randall-Sundrum model

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Abstract

We consider bulk antisymmetric tensor fields of various ranks in a Randall-Sundrum scenario. We show that, rank-2 onwards, the zero-modes of the projections of these fields on the (3+1) dimensional visible brane become increasingly weaker as the rank of the tensor increases. All such tensor fields of rank 4 or more are absent from the dynamics in four dimensions. This leaves only the zero-mode graviton to have coupling $\sim 1/M_P$ with matter, thus explaining why the large-scale behaviour of the universe is governed by gravity only. We have also computed the masses of the heavier modes upto rank-3, and shown that they are relatively less likely to have detectable accelerator signals.

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Extra spacelike dimensions with warped geometry have been shown to provide a rather elegant solution to the problem of hierarchy between the Planck scale and the electroweak scale, an embarrassment of the standard electroweak model. The exponential ‘warp factor’ attached to the Minkowski part of the metric associates a suppression factor for all masses and (gravitational) couplings of known fields residing on the ‘visible brane’ localised at one of the orbifold fixed points of the extra compact spacelike dimension. This is the essence of a Randall-Sundrum (RS) type of theory [1].

While the standard model (SM) fields are assumed to lie on a 3-brane in this scenario, the essential input is that gravity propagates in a 5-dimensional anti-de Sitter bulk spacetime [2]. A natural explanation of such a description comes from string theory, with the SM fields arising as excitation modes of an open string whose ends lie on the brane. The graviton, on the other hand, is a closed string excitation and hence its non-localisation seems to be in order [3]. Consequently, when one takes its projections on the visible brane, massless graviton mode has a coupling $\sim 1/M_P$ with all matter, while the massive modes have enhanced coupling through the warp factor. It not only accounts for the observed impact of gravity in our universe but also raises hopes for new signals in accelerator experiments [4]. However, there are various antisymmetric tensor fields which also comprise excitations of a closed string [3], and therefore can be expected to lie in the bulk similarly as gravity. The question we ask here is: can these fields also have observable effects? If not, why are the effects of their massless modes less perceptible than the force of gravitation?

Bulk fields other than gravitons have been studied earlier in RS scenarios, starting from bulk scalars which have been claimed to be required for stabilisation of the modulus [5]. Bulk gauge fields and fermions have been considered, too, with various phenomenological implications [6]. While some of such scenarios are testable in accelerator experiments [7] or observations in the neutrino sector [8], by and large they do not cause any contradiction with our observations so far.

However, the situation with tensor fields of various ranks (higher than 1) is slightly different. For example, as has been already noted, an antisymmetric rank-2 tensor field such as the Kalb-Ramond excitation [9] can be in the bulk as legitimately as the graviton, and *prima facie* has similar coupling to matter as gravity. Using a generalised form of the Einstein-Cartan action, it has been shown that such a field is equivalent to torsion in spacetime [10], on which the experimental limits are quite severe [11]. This apparent contradiction has been ameliorated in an earlier work [12] where it has been shown that the zero mode of the antisymmetric tensor field gets an additional exponential suppression compared to the graviton on the visible brane. This could well be an explanation of why we see the effect of curvature but not of torsion in the evolution of the universe. Arguments

in this line will however be complete only when we can similarly address the effects of other, higher rank, antisymmetric fields which occur in the NS-NS or RR sector of closed string excitations [13]. We address that question in the current study, in the special context of RS-like models. What we wish to point out as a whole is that the zero mode of *any antisymmetric tensor field* undergoes progressive exponential suppression increasing with the rank of the tensor. Moreover, for most higher rank tensor fields it becomes impossible to have non-vanishing components on the brane, partly because of the antisymmetric nature of the tensor, and partly due to the gauge freedom of these fields, which reduces the available degrees of freedom on the brane to zero.

In order that a rank- n antisymmetric tensor gauge field $X_{a_1 a_2 \dots a_n}$ can be part of the dynamics, one should be able to write down a rank- $(n+1)$ field strength tensor

$$Y_{a_1 a_2 \dots a_{n+1}} = \partial_{[a_{n+1}} X_{a_1 a_2 \dots a_n]} \quad (1)$$

Since a spacetime of dimension D admits of a maximum rank D for an antisymmetric tensor, one can at most have $(n+1) = D$. Thus any antisymmetric tensor field X can have a maximum rank $D-1$, beyond which it will all have either zero components or will become an auxiliary field with the field strength tensor vanishing identically. Such an auxiliary field can be eliminated via the equations of motion if it has no mass term in the bulk, a feature shared by all antisymmetric tensor excitations of a closed string due to gauge invariance.

Now let us consider a 3-brane in an RS-type 5-dimensional anti-de Sitter bulk spacetime, where the extra spatial dimension has been compactified on an S_1/Z_2 orbifold. There are two branes at the orbifold fixed points $\phi = 0$ and π , where ϕ is the angular variable corresponding to the compact dimension. In such a scenario, the 5-dimensional metric can be written as

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2 \quad (2)$$

with $\eta_{\mu\nu} = (-, +, +, +)$, and $\sigma = kr_c |\phi|$. r_c is the radius of the compact dimension y , with $y = r_c \phi$. k is on the order of the 5-dimensional Planck mass M . The standard model fields reside at $\phi = \pi$ while gravity peaks at $\phi = 0$. The dimensional parameters defined above are related to the 4-dimensional Planck scale M_P through the relation

$$M_P^2 = \frac{M^3}{k} [1 - e^{-2kr_c\pi}] \quad (3)$$

Clearly, M_P , M and k are all of the same order of magnitude. For $kr_c \simeq 12$ the exponential factor (frequently referred to as the ‘warp factor’) produces TeV scale mass parameters (of the form $m = Me^{-kr_c\pi}$) on the visible brane. Thus the hierarchy between the Planck and TeV scales is achieved without fine-tuning.

The closed string modes of excitation pertinent to such a scenario are antisymmetric tensor fields of various ranks, in addition to the graviton. Following the reasoning given above, such fields can at most be of rank-4. A rank-5 field has a rank-6 field strength tensor in the kinetic energy term, which, by virtue of its complete antisymmetry, cannot exist in 5-dimensions. Thus such a field can be removed using the equations of motion, while fields of even higher rank themselves vanish identically.

Taking a closer look at the rank-5 field strength tensor Y_{ABCMN} of a rank-4 field, one gets two kinds of terms, namely :

$$Y_{ABCMN} = \partial_{[\mu} X_{\nu\alpha\beta y]} \quad (4)$$

and

$$Y_{ABCMN} = \partial_{[y} X_{\mu\nu\alpha\beta]} \quad (5)$$

where the Latin indices denote bulk co-ordinates, the Greek indices run over the (3+1) Minkowski co-ordinates and y stands for the compact dimension. The first class of terms can be removed using the gauge freedom

$$\delta X_{ABCM} = \partial_{[A} \Lambda_{BCM]} \quad (6)$$

which allows the use of 10 gauge-fixing conditions for an antisymmetric Λ_{BCM} . As a result one can use

$$X_{\nu\alpha\beta y} = 0 \quad (7)$$

The second class of terms do not yield any kinetic energy for $X_{\mu\nu\alpha\beta}$ on the visible brane, and they can thus be removed using the equation of motion.⁴ Thus the rank-4 antisymmetric tensor fields (and of course those of all higher ranks) have no role to play in the four-dimensional world in the RS scenario.

Thus all that can matter are the lower rank antisymmetric tensor fields. The case of a rank-2 field in the bulk (known as the Kalb-Ramond field) has been already investigated, leading to the rather interesting observation that the warped geometry results in an additional exponential suppression of its zero mode on the visible brane with respect to the graviton. This suggests an explanation of why torsion can be imperceptible relative to curvature in our four-dimensional universe.

⁴In principle, such an auxiliary field can have an interaction term of the form $X_{\mu\nu\alpha\beta} B^{\mu\nu} B^{\alpha\beta}$ with second rank antisymmetric tensor fields. If such terms at all exist, they will at most result in quartic self-couplings of the rank-2 field

Let us now consider the only antisymmetric tensor field of a higher rank, surviving on the 3-brane. This is a rank three tensor X_{MNA} , with the corresponding field strength Y_{MNAB} . The action for such a field in 5-dimensions is

$$S = \int d^5x \sqrt{-G} Y_{MNAB} Y^{MNAB} \quad (8)$$

where G is the determinant of the 5-dimensional metric. Using the explicit form of the RS metric and taking into account the gauge fixing condition $X_{\mu\nu y} = 0$, one obtains

$$S_x = \int d^4x \int d\phi [e^{4\sigma} \eta^{\mu\lambda} \eta^{\nu\rho} \eta^{\alpha\gamma} \eta^{\beta\delta} Y_{\mu\nu\alpha\beta} Y_{\lambda\rho\gamma\delta} + 4 \frac{e^{2\sigma}}{r_c^2} \eta^{\mu\lambda} \eta^{\nu\rho} \eta^{\alpha\gamma} \partial_\phi X_{\mu\nu\alpha} \partial_\phi X_{\lambda\rho\gamma}] \quad (9)$$

Considering the Kaluza Klein decomposition of the field X ,

$$X_{\mu\nu\alpha}(x, \phi) = \sum_{n=0}^{\infty} X_{\mu\nu\alpha}^n(x) \frac{\chi^n(\phi)}{\sqrt{r_c}} \quad (10)$$

an effective action of the following form can be obtained in terms of the projections $X_{\mu\nu\alpha}^n$ on the visible brane:

$$S_X = \int d^4x \sum_n [\eta^{\mu\lambda} \eta^{\nu\rho} \eta^{\alpha\gamma} \eta^{\beta\delta} Y_{\mu\nu\alpha\beta}^n Y_{\lambda\rho\gamma\delta}^n + 4m_n^2 \eta^{\mu\lambda} \eta^{\nu\rho} \eta^{\alpha\gamma} X_{\mu\nu\alpha}^n X_{\lambda\rho\gamma}^n] \quad (11)$$

where m_n^2 is defined through the relation

$$-\frac{1}{r_c^2} \frac{d}{d\phi} (e^{2\sigma} \frac{d}{d\phi} \chi^n) = m_n^2 \chi^n e^{4\sigma} \quad (12)$$

and χ^n satisfies the orthonormality condition

$$\int e^{4\sigma} \chi^m(\phi) \chi^n(\phi) d\phi = \delta_{mn} \quad (13)$$

In terms of $z_n = \frac{m_n}{k} e^\sigma$ and $f_n = e^\sigma \chi^n$, equation (12) can be recast in the form

$$[z_n^2 \frac{d^2}{dz_n^2} + z_n \frac{d}{dz_n} + (z_n^2 - 1)] f_n = 0 \quad (14)$$

which is a Bessel Equation of order 1.

The solution for χ^n is given by

$$\chi^n = e^{-\sigma} f_n = \frac{e^{-\sigma}}{N_n} [J_1(z_n) + \alpha_n Y_1(z_n)] \quad (15)$$

where $J_1(z_n)$ and $Y_1(z_n)$ respectively are Bessel and Neumann functions of order 1. α_n and N_n are integration constants which can be determined from orthogonality and the continuity conditions at the orbifold fixed points. In addition, the continuity condition for the derivative of χ_n at $\phi = 0$ yields

$$\alpha_n = -\frac{J_2(\frac{m_n}{k})}{Y_2(\frac{m_n}{k})} \quad (16)$$

Using the fact that $e^{kr_c\pi} \gg 1$ and the mass values m_n on the brane is expected to be on the order of TeV scale ($\ll k$),

$$\alpha_n \sim \frac{\pi}{2^5} \left(\frac{m_n}{k}\right)^4 \ll 1 \quad (17)$$

The boundary condition at $\phi = \pi$ gives

$$J_2(x_n) = 0 \quad (18)$$

where $x_n = z_n(\pi) = \frac{m_n}{k} e^{kr_c\pi}$. The roots of the above equation determine the masses of the higher excitation modes. As x_n is of order unity, the massive modes lie in the TeV scale.

Furthermore, the normalisation condition yields

$$N_n = \frac{e^{kr_c\pi}}{\sqrt{kr_c}} J_1(x_n) \quad (19)$$

and the massive modes can be obtained from the equation

$$\chi^n(z_n) = \sqrt{kr_c} \frac{e^\sigma}{e^{kr_c\pi}} \frac{J_1(z_n)}{J_1(x_n)} \quad (20)$$

The values of the first few massive modes of the rank-3 antisymmetric tensor field are listed in Table 1, where we have also shown the masses of the graviton as well as the rank-2 antisymmetric Kaluza-Klein modes. It can be noticed that the rank-3 field has higher mass than the remaining two at every order, and, while the Kalb-Ramond massive modes can have some signature at, say, the Large hadron collider (LHC), that of the rank-3 massive tensor field is likely to be more elusive.

n	1	2	3	4
$m_n^{grav} (TeV)$	1.66	3.04	4.40	5.77
$m_n^{KR} (TeV)$	2.87	5.26	7.62	9.99
$m_n^X (TeV)$	4.44	7.28	10.05	12.79

Table 1: The masses of a few low-lying modes of the graviton, Kalb-Ramond (KR) and rank-3 antisymmetric tensor (X) fields, for $kr_c = 12$ and $k = 10^{19} GeV$.

Finally, and most crucially, we examine the massless mode, whose strength on the brane needs to be compared to that of the graviton and the rank-2 field. The solution for this mode is given by

$$\chi_0 = -\frac{C_1}{2kr_c} e^{-2\sigma} + C_2 \quad (21)$$

Requiring the continuity of $\frac{d\chi^0}{d\phi}$ at $\phi = \pi$, one obtains $C_1 = 0$. The normalisation condition finally gives

$$\chi^0 = \sqrt{2kr_c} e^{-2kr_c\pi} \quad (22)$$

This shows that the zero mode of the rank-3 antisymmetric tensor field is suppressed by an additional exponential factor relative to the corresponding rank-2 field which already has an exponential suppression compared to the zero mode of the graviton. Using the same argument as in reference [12], one can translate this result into the coupling of the field X to matter, and show that the interaction with, say, spin-1/2 fields is suppressed by a factor $e^{-2kr_c\pi}$. Thus the higher order antisymmetric field excitations have progressively insignificant roles to play on the visible brane, with the fields vanishing identically beyond rank 3.

In conclusion, the graviton seems to have a unique role among the various closed string excitations in a warped geometry. This is because the intensity of its zero mode on the 3-brane leads to coupling $\sim 1/M_P$ with matter fields, which is consistent with the part played by gravity (or more precisely the curvature of spacetime) observed in our universe. On the contrary, while bulk antisymmetric tensor fields upto rank-3 can still have non-vanishing zero modes in four-dimensional spacetime, their strength is progressively diminished for ranks-2 and 3. This may well serve as an explanation of why the evolution of our universe is solely controlled by gravitation. In addition, the masses of the higher modes also tend to increase with rank, making them less and less relevant to accelerator experiments.

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